date the use of the Euler-Bernoulli theory in determining flexural stresses, even in the case of reasonably high impact.

It might also be of interest to note that in general, the dependence of the response on the slenderness ratio was one of increased oscillatory motion with higher L/r.

An extension of these results to the limiting cases of rigid supports and large slenderness ratios presented some mathematical problems. The difficulties, considering the Timoshenko model, involve poor convergence of the series solutions and rapid oscillations in the vicinity of the wave fronts. However, the method of singular perturbation and the use of the theory of characteristics can be used to relieve the problem. Results of this analysis will be represented in a future publication.

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Steady-State Burning of Double-Base Propellants at Low Pressures

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Nomenclature

A = cross-sectional area of propellant
c = heat capacity of propellant
E = energy transmitted to solid
k = thermal conductivity of propellant
r = linear burning rate
T = temperature
y = coordinate parallel to the axis of propellant; the burning surface at $y = 0$
ρ = mass density of propellant
0 = initial state
p = propellant
s = surface

Introduction

One of the difficulties in constructing a mathematical model to describe the deflagration characteristics of double-base propellants is the lack of understanding of the role of each parameter. There has not been a sufficient number of critical experiments on double-base propellants to characterize its combustion mechanism. The extent of the so-called solid phase (or condensed phase) reactions and the surface reaction, the mechanism of decomposition, the magnitudes and effects of heat transfer from the gas phase to the solid phase, the influence of the chemical reaction at the surface on the burning rate, and the mechanism of the gas-phase reaction are not clarified yet. This Note deals with the burning rate of M-2 double-base propellant at various initial temperatures and at low pressures ranging from 2.4 to 165 psia.

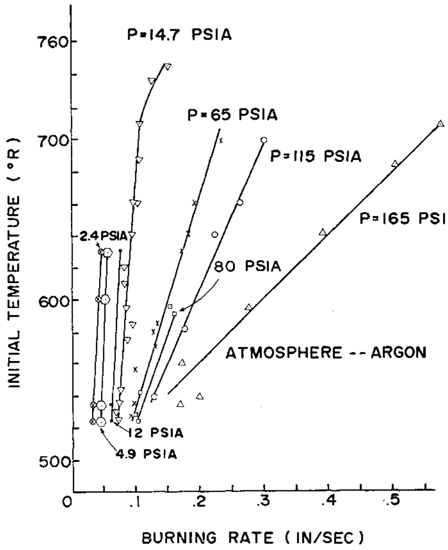

Fig. 1 Experimental results...burning rate vs initial temperature at various pressures (in argon).

The deflagration mechanism of double-base propellants has been investigated by many researchers in the past. Heller and Gordon,[1] Rice and Ginelli,[2] and Parr and Crawford[3] investigated the reactions and existence of "foam," "fizz," "dark," and "flame" zones. Wilfong et al.[4] also investigated the burning rate as a function of the chamber pressure and initial temperature. Recently there have been many publications coming out of Russian[5,6] research on double-base propellants.

Experimental Apparatus and Procedure

The experimental apparatus was arranged to accommodate ignition of the propellant in a gaseous atmosphere at low- and high-ambient pressures and low- and high-initial propellant temperatures. The combustion chamber was made of stainless steel. The propellant was placed vertically in the chamber and was ignited at the lower end by a heater. The burning rate was determined using a fuse wire technique. Leads were connected through the chamber so that the burning ing rate and temperature of the propellant could be recorded. A quartz window was placed in one end of the chamber in order to observe the burning process.

The chamber was filled with either argon or air during the experiment. The gases entering the chamber were either heated or cooled before entering the chamber in order to raise or lower the initial propellant temperature. The initial temperature of the propellant was measured by placing an iron-constantan thermocouple through a hole in the propellant. The physical properties of M-2 double-base propellant are given by Suh et al.[7] The propellant was coated in order to insure a cigarette-type burning. The coated propellant was dried for 5 hr at 160°F.

Experimental Results

Burning rate measurements

Figure 1 shows the plot of the burning rate vs the initial temperature of the propellant obtained in argon at pressures ranging from 2.4 to 165 psia. It should be noted that the curves of the experimental results obtained at 14.7 psia or higher tend to cross each other at about 520°R. The extrapolation of these curves indicates that when the initial tem-

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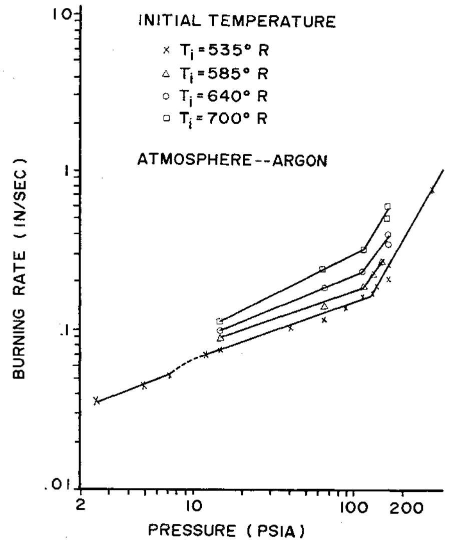

Fig. 2 Experimental results...burning rate vs pressure at various initial temperatures (in argon).

perature of the propellant is less than 520°R, the increase in pressure has inverse effects on the burning rate. It is found that this initial temperature of 520°R corresponds to the self-extinction temperature, as discussed later in this paper. As the initial temperature is increased, the auto-ignition temperature is approached and self-deflagration takes place.

When the ambient pressure is subatmospheric, the increase in the initial temperature yields little increase in the burning rate. However, even in this pressure range, the burning rate is a sensitive function of the ambient pressure. It should be noted that the experimental curves obtained at 14.7 psia and below are all nearly parallel. Also, the curves for 65, 80, and 115 psia are almost parallel, although the slope of the curve for 65 psia is quite different from that for 14.7 psia. A drastic change in the slope of the curves occurs at about 165 psia. Some of these features are shown more clearly in Fig. 2.

Figure 2 shows the same experimental results obtained in argon plotted for the burning rate as a function of pressure at various initial temperatures. In addition to the normally expected results that show the dependence of the burning rate on the pressure, it should be noticed that there is a sharp change in the burning rate at about 115 psia. The sharp change was observed to coincide with the first appearance of a flame, i.e., below 115 psia there was no flame. Most of the work done in the past has been performed at much higher pressures than those used in this experiment. Therefore, this interesting observation appears not to have been reported. A less drastic change seems to occur at about 14.7 psia. Several of the data points shown in Figs. 1, 2, and 3 are average values of several experimental results. This was done when the experimental points all fell close to each other.

Figure 3 shows similar results obtained in air when the intial temperature of the propellant was 535°R. In this case, the sharp change in the slope of the burning rate curve occurs at about 45 psia.

Self-extinction of burning

As discussed briefly earlier, another phenomenon associated with deflagration at relatively low pressure and temperature was self-extinction of burning. After the propellant was ignited at low temperatures ranging from 520 to 525°R, extinction occurred at all pressures used in this experiment. Below 115 psia extinction occurred at 520°R (15.5°C) and above 115 psia it occurred at 525°R (18.3°C). Extinction occurred either, 1) before the burning surface reached any of the three wires that were placed through the propellant, 2) after burning through any of the wires, or 3) when the burning surface reached any of the wires. In the case of 1) and 3), a substantial portion of thermal energy might have been absorbed by the wires to cause extinction.

Since pure nitroglycerin solidifies at 13°C, it was suspected that the energy being absorbed to liquify the nitroglycerin might have been responsible for self-extinction. However, when the endotherm of the propellant was checked using a differential scanning calorimeter, there were no endotherms or exotherms (Kirby and Suh[8]). This could be because of the fact that the nitroglycerin molecules are well dispersed in the propellant between the long nitrocellulose molecules as plasticizers forming a solution.

Discussion of the Experimental Results

The fact that the self-extinction occurs near 15.5°C (520°R) may be used to estimate the minimum thermal energy that needs to be transmitted into the solid phase at the surface to sustain the steady-state burning of the propellants. The condition necessary for self-sustaining steady-state burning, in the absence of any external heat flux and solid-phase reaction, is that the volume fraction of the thermal energy generated at the surface that is transmitted into the solid E must be equal to the heat conduction per unit volume into the solid required to maintain the existing steady-state temperature profile in the solid, i.e.,

$$EAr = -kA(\partial T_p/\partial y) \qquad (1)$$

Since

$$T_p = (T_s - T_0)\exp(-\rho c r y/k) + T_0 \qquad (2)$$

Equation (2) yields

$$E = \rho c(T_s - T_0) \qquad (3)$$

Suh et al.[7] have shown that the surface temperature is about 300–320°C at pressures of 5 and 10 psia. The surface temperature is predicted considering the thermocouple response characteristics in propellants. Taking $c = 0.37$ cal/g − °K, $\rho = 1.6$ g/cm^3, $T_s = 315$°C, and $T_0 = 15.5$°C, E becomes 178 cal/cm^3. Therefore, the minimum amount of energy that has to be supplied in the solid, either internally or externally or both, may be very close to 178 cal/cm^3. To date there is no definitive work done that can give the exact magnitude of the heat released internally. Furthermore, the physical significance of the solid-phase reaction is not clear. It seems that since any reaction is a consequence of forming or breaking chemical bonds and since energy is highest at surfaces, the solid- or condensed-phase reaction must occur at surfaces or interfaces. The surfaces can exist internally as well as ex-

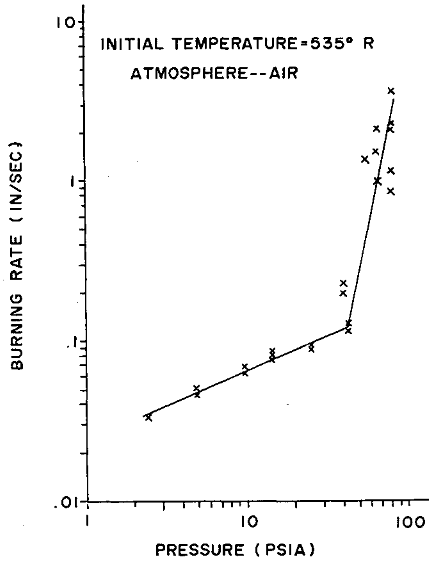

Fig. 3 Experimental results...burning rate vs pressure in air.

ternally, the internal surfaces being due to the existence of midro-voids, impurities, and chemical additives. Therefore, the solid phase reaction should depend a great deal on the physical state of the propellant as well as the chemical nature. Of course, this argument does not preclude the fact that some of the bonds away from the surface can break; rather it implies that the activation energy for bond breaking at the surface must be lower than that away from the surface.

The abrupt change in the burning rate at certain pressures is shown to be due to the fact that discrete gas phase reactions occurring a few microns above the burning surface reach the surface at these pressures (Thompson and Suh[9]). The flame seems to appear when the concentration of reactants increases sufficiently because of the increased burning rate.

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Mobility of NO+ Ions in Air

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Nomenclature

e = electron charge
f_j = fractional concentration of the j species
k = Boltzmann constant
m_j = mass of particle of the j species
n = total number density
n_o = Loschmidt number
r_o = nondimensionalizing length in the inverse power repulsive potential
r_{ij} = interparticle separation
$A^{(1)}(\nu)$ = collision integral for an inverse power potential
$B(\nu)$ = collision integral correcting for a weak secondary potential
K_{ij} = coefficient of the short-range inverse power repulsive potential
T = temperature
α_j = polarizability of the j species
μ_{ij} = mobility of ion i in the j species
ν = index of the inverse power repulsive potential
σ_{ij} = hard sphere diameter
σ_{eff} = effective hard sphere diameter of the inverse power potential
$\psi(r_{ij})$ = interparticle potential
$\Gamma(x)$ = gamma function
$\Omega_{ij}^{(1,1)}$ = collision integral characteristic of diffusion

Introduction

IN order to use the results of any theory of continuum electrostatic probe operation, it is necessary to possess a good knowledge of the mobility of the particular ions being collected. In air plasmas, the ions of most interest are NO^+. In spite of the importance of this ion species, there are few experimental data available on its mobility at room temperature, and none at all at the higher plasma temperatures of interest in probe diagnostics. An alternative to direct measurement of the mobility is an accurate calculation from known or measured interparticle potential functions such as that based on the Chapman-Enskog theory, for example. This is the approach taken in this note, since reasonable hypotheses may be made regarding the interaction potential of NO^+ and the air species.

In the first approximation of the Chapman-Enskog theory,[1] the mobility of an ion (subscript i) in a gas of neutral particles (subscript j) may be written

$$\mu_{ij} = \frac{3}{16}\left(\frac{m_i + m_j}{m_i m_j}\right)\frac{e}{n(1/\Omega_{ij}^{(1,1)})} \tag{1}$$

where $n = n_i + n_j$, and $\Omega_{ij}^{(1,1)}$ is the collision integral appropriate to diffusion.[1,2] The collision integral depends in a complicated way on the interparticle potential and, for a given pair of particles, is a function of the temperature only. We shall assume the potential to be the sum of a short-range repulsive potential and an attractive polarization potential

$$\psi(r_{ij}) = \psi_{\text{rep}}(r_{ij}) + \psi_{\text{att}}(r_{ij}) \tag{2}$$

$$\psi_{\text{rep}}(r_{ij}) = K_{ij}(r_o/r_{ij})^{\nu} \tag{3}$$

$$\psi_{\text{att}}(r_{ij}) = -\alpha_j e^2/2r_{ij}^4 \tag{4}$$

In general, it will be true that the repulsive potential will depend on a higher inverse power of the interparticle distance than the attractive potential (i.e. $\nu > 4$) so that the repulsive potential will dominate at high temperatures and the polarization potential should dominate at low temperatures. In the special case when the repulsive potential is represented by a hard sphere core ($\nu \to \infty$), the collision integral has been calculated by Langevin and later by Hassé. A function that for this case is closely related to $\Omega_{ij}^{(1,1)}$ is tabulated by McDaniel[3] where references to the original calculations are listed. This calculation is useful at low temperatures where the exact form of the repulsive potential is least important. The form of the potential function Eqs. (2–4) is in fact suggested by the success of the Langevin-Hassé theory in predicting the mobility in many cases.[3] It should be pointed out, however, that while this potential is assumed to be suitable for the interaction of NO^+ and the air species, it may be inadequate in other cases where quadrupole and dipole-dipole (dispersion) forces interactions may contribute substantially to the attractive portion of the potential function.

The exact calculation of $\Omega_{ij}^{(1,1)}$ for potentials of the type given by Eqs. (2–4) is very complicated, except for special cases.[1,2] Moreover, such a complete calculation is not warranted since the measured repulsive potentials on which it would be based are known only over restricted temperature ranges (e.g. Ref. 4). At high temperatures, where the attractive potential is weak compared to the repulsive potential, Chapman and Cowling[2] show that for the type of interparticle potential assumed here, the collision integral can be

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